|  |  |
| --- | --- |
| **Unit** | **G2** |
| **Title** | Congruence |
| **Target Standards** | **G.CO.5, G.CO.6, G.CO.7, G.CO.8, G.CO.9, G.CO.10, G.CO.11** |
| **Mathematical Goals** | Students will…   * Specify sequences of rigid motions that will carry a figure onto another G.CO.5 * Understand that there is more that one sequence of rigid motions that will carry a figure onto another G.CO.5 * Use the definition of congruence in terms of rigid motions to decide if two figures are congruent or not G.CO.6 * Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC) G.CO.7 * Can explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions G.CO.8 * Prove theorems about lines and angles G.CO.9 * Prove theorems about parallelograms G.CO.11 * Prove base angles of isosceles triangles are congruent G.CO.10 |
| ***The story before this unit (including prior knowledge)*** | In the G1 unit, students built off of their middle school geometry foundations; students made formal geometric constructions and developed formal definitions for rigid motions and several key geometric terms. Students completed single rigid motions (translations, rotations, reflections) and described rotations and reflections that carry polygons onto themselves. |
| ***The part of the story happening in this unit*** | In this unit, students build on their understanding of rigid motions to develop an understanding of congruence. Students use the definition of congruence in terms of rigid motions to prove several theorems about lines, angles, triangles and parallelograms. |
| ***The story after this unit*** | In the G3 unit students will extend their knowledge of transformations to include the non-rigid dilation. Students will explore properties of dilations and expand that knowledge to understand similarity in triangles and other shapes and use this understanding to solve problems. |

**UNIT FLOW SUMMARY**

|  |  |
| --- | --- |
| **UNIT G2** (11 - 15 days) | **Congruence** |
| **Section 0** (1 day) | **Diagnostic Pre-Unit Assessment** |
| **Section 1** (2 days) | **Sequences of Rigid Motions (hook/umbrella activity)** |
| **Section 2** (1 -2 days) | **Defining Congruence** |
| **Section 3** (2 - 3 days) | **Triangle Congruence** |
| **Section 4** (3 - 4 days) | **Proving Congruence** |
| **Section 5** (1 day) | **Summative Assessment** |

|  |  |
| --- | --- |
| **Section 0:** 1 day | **Diagnostic Pre-assessment** |
| **Pre-Unit Assessment Targets** | Assess students’ ability to   * Make formal geometric constructions (parallel and perpendicular lines) [G.CO.12] * Construct a square and explain why the construction yields such [G.CO.13] * Given a geometric figure and a rotation, reflection or translation draw the transformed figure [G.CO.5] * Understand and explain the formal definition of reflection [G.CO.4]   \*Much of this pre-assessment is the same as the summative assessment from G1 |
| **Sample Activity** | * [*Link to assessment*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsclgwR3FGMDRzWTg/edit) |



|  |  |
| --- | --- |
| **Section 1:** 2 days | **Sequences of Rigid Motions (hook/umbrella activity)** |
| **Mathematical Goal** | * Students can specify sequences of rigid motions that will carry a figure onto another * Students explore different ways to transform one figure onto another * After completing a sequence of rigid motions, students look to see if there is another way to carry a figure to another in less steps |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, students begin to explore sequences of rigid motions. Previously, students have worked with single rigid motions and here they begin to perform multiple rigid motions to a single object. This section is meant to help students develop an understanding of sequences of rigid motions before moving to the concrete application seen when proving congruence through rigid motions that they will see later in this unit. |
| **Sample Activity 1.1** | [*Sequences of Rigid Motions*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsMEJGZTBqZ09jaHM/edit), High Tech High, Jade White  **WHAT:** In this activity, students begin to explore sequences of transformations that will carry a figure onto another G.CO.5. Students are given two figures and are asked to find a sequence which will carry a figure to another. Students are then asked to find a second sequence or possibly a single transformation that also moves the first figure to the second.  **WHY:** The purpose of this activity is to allow students to build on their understandings of single rigid motions before moving directly into congruence. Here, students must attend to precision MP.6 in their transformations and must also construct viable arguments MP.3 when explaining how a certain sequence of transformations can be completed in fewer steps. Additionally, students must use tools appropriately MP.5 to help them precisely complete the sequence of rigid motions. |
| **Sample Activity 1.2** | [*Sequences of Rigid Motions 2*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsUWtpT2hmaUpGclE/edit), U of A/High Tech High, Jade White  **WHAT:** In this activity, students further explore sequences of transformations that will carry a figure onto another G.CO.5. In the first two parts of this activity, students first reflect an object over two different lines then explore how a single rotation or translation can carry out the same movement. In the last part, students explore various possibilities, some single and some double transformations, to move figure A to figure B.  **WHY:** Before learning the definition of congruence through rigid motions, it is important that students are familiar with sequences of rigid motions and the idea that there can be multiple ways to carry a given figure onto another. The activity helps students to look for and express regularity in repeated reasoning MP.8 in carrying out sequences of rigid transformations. Additionally, students must attend to precision MP.6 when completing the transformations to ensure that when comparing different possible sequences the transformed figure is in fact in the correct location. |
| **Target Standards** | G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. |
| **Mathematical Practices** | MP.3, MP.5, MP.6, MP.8 |

|  |  |
| --- | --- |
| **Section 2:** 1 - 2 days | **Defining Congruence** |
| **Mathematical Goals** | * Students understand the definition of congruence in terms of rigid motions * Students can use the definition of congruence in terms of rigid motions to decide if two figures are congruent or not |
| **Narrative overview of section**  (and how the standards are achieved) | This section serves as transition days between rigid motions and proving congruence. Here students first see the definition of congruence and then look at previous problems they have completed but now through the lens of congruence. In later sections, students will refine and expand on their understanding of congruence, here they first see and learn what it is. |
| **Sample Activity 2.1** | [Congruence through Rigid Motions](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsVzlfZG9JTkNiWDA/edit), High Tech High, Jade White  **WHAT:** In this activity, students examine a sequence of rigid motions to show that two triangles are congruent. The first two parts of this activity were seen above in [*Sequences of Rigid Motions*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsMEJGZTBqZ09jaHM/edit). The difference here, is in the wording of the activities where students are now asked to show that two triangles are congruent through a sequence of rigid motions G.CO.6. The final questions in this activity provide a launching pad for going more in-depth into what is necessary to prove that two triangles are congruent.  **WHY:** The purpose of this activity is to build students understanding of congruence through sequences of rigid motions. With their experience in showing and explaining sequences of rigid motions, students should begin to look for and express regularity in repeated reasoning MP.8 through showing congruence in these problems. This activity also provides the foundation for section 3 where students will begin to explore CPCTC and criteria for triangle congruence (ASA, SAS, and SSS). |
| **Sample Activity 2.2** | [*Understanding Triangle Congruence*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsTWRuMU5RVjRlZTg/edit), High Tech High, Jade White  **WHAT:** In the first part of this activity, students are given three triangles and they must construct a congruent triangle. In the second part of this activity, students are again given three triangles but this time the goal is to construct a triangle that appears congruent but actually is not; this could be proven through a sequence of rigid motions G.CO.6. The third part of the activity has students share their constructed triangles from parts one and two with a partner to examine the similarities and differences and how this relates to congruence.  **WHY:** The purpose of this activity is to give students an opportunity to understanding what it takes to create/construct a congruent or non-congruent figure MP.4. Students must attend to precision MP.6 and use tools strategically MP.5 in their constructions to ensure their congruent triangles are in fact congruent. Part three of this activity allows students a chance to critique the reasoning of others and to construct viable arguments about their observations with congruent triangles MP.3. Additionally, in section 3 of this unit, students will develop more formal understandings of triangle congruence; this activity is designed to help students develop initial conceptual understanding of congruence. |
| **Target Standards** | G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. |
| **Mathematical Practices** | MP.3, MP.4, MP.5, MP.6, MP.8 |

|  |  |
| --- | --- |
| **Section 3:** 2 - 3 days | **Triangle Congruence** |
| **Mathematical Goals** | * Students use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC) * Students can explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions * Students can prove that base angles of isosceles triangles are congruent |
| **Narrative overview of section**  (and how the standards are achieved) | In the previous section, students were introduced to the idea of congruence through rigid motions. Students constructed both congruent and non-congruent figures and began to make observations about congruent triangles. In this section, students formalize their understanding of congruent triangles. Students understand that two triangles are congruent if and only if corresponding pairs of sides and corresponding angles are congruent (CPCTC). Students also understand how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions. Lastly, students can prove that base angles of isosceles triangles are congruent. |
| **Sample Activity 3.1** | [*Properties of Congruent Triangles*](https://www.illustrativemathematics.org/illustrations/1637)*,* Illustrative Mathematics  **WHAT:** In this activity, students are given two triangles and are told that through a sequence of rigid motions, you can map triangle ABC to triangle DEF. The exploration questions ask students about the relationships between the sides and angles of the two given triangles.  **WHY:** At this point, students have experience with sequences of rigid motions and what congruence is. This problem extends those ideas to begin to build the understanding of properties that congruent triangles have. As explained on the IM website, “The goal of this task is to understand how congruence of triangles, defined in terms of rigid motions, relates to the corresponding sides and angles of these triangles. In particular, there is a sequence of rigid motions mapping one triangle to another if and only if these two triangles have congruent corresponding sides and angles.” G.CO.7 Additionally, students will need to reason abstractly and quantitatively MP.2 while completing this problem; working between the sequence of rigid motions to map one triangle to the other while keeping track of the different parts of the triangles and how they are related. |
| **Sample Activity 3.2** | [*Congruence Theorems*](http://illuminations.nctm.org/Activity.aspx?id=3504), NCTM Illuminations  **WHAT:** In this activity, students choose 3 elements to make a triangle. If students can create two distinct triangles using the same 3 parts, then they can conclude that those parts do not prove congruence. Students are working towards understanding the criteria necessary for triangle congruence G.CO.8. Through this activity, students also see that two triangles are congruent if and only if corresponding pairs of sides and corresponding angles are congruent (CPCTC) G.CO.7.  **WHY:** The purpose of this activity is to provide students an opportunity to experiment with different arrangements of triangle parts (sides and angles) to see what is necessary for triangle congruence. The goal is that after testing all possible arrangements, students will be able to explain which arrangements provide triangle congruence and which do not. Through the testing phase, students must persevere through all possible arrangements MP.1 and should begin to look for repeated reasoning through the different trials MP.8. |
| **Sample Activity 3.3** | [*Analyzing Congruency Proofs*](http://map.mathshell.org/materials/download.php?fileid=1302), MARS  **WHAT:** In the first part of this activity, students are given conditions about triangles (angle measures/side lengths) and told that two triangles with these properties will be congruent G.CO.7. Students must explain whether this is true or not both visually, through a drawing, and written MP.5. The second part of the activity provides students with information about two triangles and using only this information, students decide if the triangles are congruent or not G.CO.8.  **WHY:** At this point in the unit, students should be familiar with both CPCTC and criteria for triangle congruence from the previous sample activities. In their explanations about if the given triangles are congruent or not, students must construct viable arguments to support their ideas MP.3. Through this activity and repeated exposure to different triangle criteria, students should be able to look for and make use of structure in identifying whether triangles are congruent or not MP.7. |
| **Sample Activity 3.4** | [*Why does SAS Work?*](https://www.illustrativemathematics.org/illustrations/109)*,* [*Why does ASA work?*](https://www.illustrativemathematics.org/illustrations/339)*,* [*When does SSA work to determine triangle congruence?*](https://www.illustrativemathematics.org/illustrations/340)*,* [*Why does SSS work?*](https://www.illustrativemathematics.org/illustrations/110)*, Illustrative Mathematics*  **WHAT:** In these activities, students look at specific triangle criteria (SAS, ASA, SSA, SSS) through sequences of rigid motions to see how they work.  **WHY:** These four tasks are grouped together because they all show triangle congruence, each spotlighting one of the criteria for triangle congruence G.CO.8. In the above activities, students were exposed to the different criteria so some or all of the activities here can be used as needed to ensure that students understand the different criteria. Because students have seen these ideas in the previous activities, this is a great place for students to look for and express regularity in repeated reasoning through these ideas MP.8. |
| **Sample Activity 3.5** | [*Angle bisection and midpoints of line segments*](https://www.illustrativemathematics.org/illustrations/1320)*,* Illustrative Mathematics  **WHAT:** In this task, students explore the relationship between an angle bisection and a line bisection (midpoint)  **WHY:** This problem allows students to use the constructions they know along with criteria for triangle congruence G.CO.8 to prove bisectors and midpoints in the given activity. Students should be able to model with precision MP.6 the angle and line bisection constructions needed to represent the conditions presented in this task. |
| **Sample Activity 3.6** | [*Isosceles Triangle Theorem*](https://www.illustrativemathematics.org/illustrations/1921)*,* Illustrative Mathematics  **WHAT:** In this activity students prove that if two sides of a triangle are congruent then their opposite angles are congruent as well G.CO.10. Students should be able to use the ideas of congruent triangles above to prove this specific feature of the isosceles triangle.  **WHY:** This task provides students with an opportunity to use their understandings about triangle congruence in a very specific case; the base angles of an isosceles triangle are congruent. |
| **Target Standards** | G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.  G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.  G.CO.10 Prove theorems about triangles. Theorems include: ~~measures of interior angles of a triangle sum to 180°~~; base angles of isosceles triangles are congruent; ~~the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point~~. |
| **Mathematical Practices** | MP.1, MP.2, MP.3, MP.5, MP.7, MP.8 |

|  |  |
| --- | --- |
| **Section 4:** 3 - 4 days | **Proving Congruence** |
| **Mathematical Goals** | * Students can prove theorems about lines and angles * Students can prove theorems about parallelograms |
| **Narrative overview of section**  (and how the standards are achieved) | So far in this unit, the focus has been on triangle congruence. Students should be familiar with the language of and ideas behind proving congruence. In this section, students apply their understanding of proving congruence to things other than triangles, including theorems about lines, angles and parallelograms. |
| **Sample Activity 4.1** | [*Angles Between Intersecting and Parallel Lines*](https://www.khanacademy.org/math/geometry/parallel-and-perpendicular-lines/ang_intro/v/angles-at-the-intersection-of-two-lines), Khan Academy  **WHAT:** In this section, several videos and practice questions are provided. Through these activities, students learn that vertical angles are congruent and when a transversal crosses parallel lines, both alternate interior angles and corresponding angles are congruent G.CO.9.  **WHY:** The purpose of this section is to provide students with several examples of congruent angles in given contexts. Depending on where students are and how much practice they need, teachers can either work through all of the provided examples and videos or only those that they see best fit for their students. Through the various examples and proofs, students should begin to look for and make use of structure relating to a transversal crossed by parallel lines and vertical angles MP.7.  Note: In the video Angles formed by parallel lines and transversals, it is stated that corresponding angles are congruent based on visual evidence. The teacher may want to add on to this using the parallel postulate. |
| **Sample Activity 4.2** | [*Points Equidistant from a two points in the plane*](https://www.illustrativemathematics.org/illustrations/967), Illustrative Mathematics  **WHAT:** In this activity, students explore the distance of a point on a perpendicular bisector to the two endpoints on the original line segment. Through this activity, students prove that points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints G.CO.9.  **WHY:** Students should be comfortable with perpendicular bisectors so the open nature of this problem allows students to explore the distance between the endpoints of the line segment to points on the perpendicular line in a familiar context as the build to the understanding of the above proof. This provides a good opportunity for students to engage in MP.3 “Construct Viable Arguments and Critique the Reasoning of Others.'' Also, students working on this task have multiple opportunities to engage in MP.5 ''Use Appropriate Tools Strategically'' as the task makes use of geogebra. |
| **Sample Activity 4.3** | [Quadrilaterals](https://www.khanacademy.org/math/geometry/quadrilaterals-and-polygons), Khan Academy  **WHAT:** This activity explores definitions, conditions and proofs relating to quadrilaterals. Specifically, there are videos provided to show that opposite sides and opposite angles of a parallelogram are congruent as well as that diagonals of a parallelogram bisect each other G.CO.11.  **WHY:** The purpose of this activity is to provide students with an introduction to the congruence properties found within a parallelogram. |
| **Sample Activity 4.4** | [Congruence of Parallelograms](https://www.illustrativemathematics.org/illustrations/1517), Illustrative Mathematics  **WHAT:** In this activity, students analyze parallelograms to see what criteria needs to be set in order to conclude that two parallelograms are congruent G.CO.11.  **WHY:**  The purpose of this activity is to connect students’ prior learnings about proving congruence in triangles and applying this to parallelograms. Students can also use their understandings from the previous activity ([Quadrilaterals](https://www.khanacademy.org/math/geometry/quadrilaterals-and-polygons)) as they explore different criteria of parallelograms. As stated on the IM website “This task is ideal for hands-on work or work with a computer to help visualize the possibilities….This task would be ideally suited for group work since it is open ended and calls for experimentation. Thus it provides a good opportunity for students to engage in MP.3 “Construct Viable Arguments and Critique the Reasoning of Others.'' Also, students working on this task have multiple opportunities to engage in MP.5 ''Use Appropriate Tools Strategically'' as they can use manipulatives or computer software to experiment with constructing different parallelograms.” |
| **Target Standards** | G.CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.  G.CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. |
| **Mathematical Practices** | MP.3, MP.5, MP.7 |

|  |  |
| --- | --- |
| **Section 5:** 1 day | **Summative Assessment** |
| **Pre-Unit Assessment Targets** | Assess students’ ability to   * show and explain sequences of rigid motions and analyze different sequences to move the same shape to another [G.CO.5] * explain the criteria for triangle congruence and can explain if given triangles are congruent or not [G.CO.6] [G.CO.7] [G.CO.8] * understand and explain that opposite sides of a parallelogram are congruent [G.CO.11] * explain that vertical angles are congruent [G.CO.9] * explain that when a transversal crosses parallel lines, alternate interior and corresponding angles are congruent [G.CO.9] |
| **Sample Activity** | *Possible tasks for Assessment:*   * [Showing a triangle congruence: a particular case](https://www.illustrativemathematics.org/illustrations/1547) * [*Are the Triangles Congruent*](https://www.illustrativemathematics.org/illustrations/33)(This task can be approached both by geometric theorems or rigid motions or a combination of both) * [*Midpoints of the Sides of a Parallelogram*](https://www.illustrativemathematics.org/illustrations/35)  [*Congruent angles made by parallel lines and a transverse*](https://www.illustrativemathematics.org/illustrations/1922) |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1.1 | 1.2 | 2.1 | 2.2 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 4.1 | 4.2 | 4.3 | 4.4 |
| G.CO.5 | **X** | **X** |  |  |  |  |  |  |  |  |  |  |  |  |
| G.CO.6 |  |  | **X** | **X** |  |  |  |  |  |  |  |  |  |  |
| G.CO.7 |  |  |  |  | **X** | **X** | **X** |  |  |  |  |  |  |  |
| G.CO.8 |  |  |  |  |  | **X** | **X** | **X** | **X** |  |  |  |  |  |
| G.CO.9 |  |  |  |  |  |  |  |  |  |  | **X** | **X** |  |  |
| G.CO.10 |  |  |  |  |  |  |  |  |  | **X** |  |  |  |  |
| G.CO.11 |  |  |  |  |  |  |  |  |  |  |  |  | **X** | **X** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1.1 | 1.2 | 2.1 | 2.2 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 4.1 | 4.2 | 4.3 | 4.4 |
| MP.1 |  |  |  |  |  | **X** |  |  |  |  |  |  |  |  |
| MP.2 |  |  |  |  | **X** |  |  |  |  |  |  |  |  |  |
| MP.3 | **X** |  |  | **X** |  |  | **X** |  |  |  |  | **X** |  | **X** |
| MP.4 |  |  |  | **X** |  |  |  |  |  |  |  |  |  |  |
| MP.5 | **X** |  |  | **X** |  |  | **X** |  |  |  |  | **X** |  | **X** |
| MP.6 | **X** | **X** |  | **X** |  |  |  |  | **X** |  |  |  |  |  |
| MP.7 |  |  |  |  |  |  | **X** |  |  |  | **X** |  |  |  |
| MP.8 |  | **X** | **X** |  |  | **X** |  | **X** |  |  |  |  |  |  |