|  |  |
| --- | --- |
| **Unit** | **F1** |
| **Title** | Functions |
|  **Target Standards** | **F.IF.1, F.IF.2, F.IF.3, F.IF.4, F.IF.5, F.IF.6, F.IF.7, F.IF.7.b, F.IF.7.d, F.BF.4**, **F.BF.4.a, F.LE.2** |
| **Mathematical Goals** | Students will…* Interpret key features of graphs in terms of the quantities F.IF.4
* Sketch graphs showing key features of the graph by hand and using technology F.IF.7
* Understand that a function from one set to another set assigns to each element of the domain exactly one element of the range F.IF.1
* Understand function notation F.IF.2
* Interpret statements that use function notation in various contexts F.IF.2
* Recognize that sequences are functions F.IF.3
* Recognize that sequences can sometimes be defined recursively F.IF.3
* Recognize and understand piecewise functions, step functions and absolute value functions F.IF.7b
* Relate the domain of a function to its graph F.IF.5
* Relate the domain to the quantitative relationship it describes F.IF.5
* Calculate and interpret the average rate of change of a function over a specified interval F.IF.6
* Estimate the rate of change from a graph F.IF.6
* Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.7.d
* Find the inverse function of basic functions F.BF.4
* Write an expression for the inverse F.BF.4a
 |
| ***The story before this unit (including prior knowledge)*** | In 8th grade, students are first introduced to the notion of functions. Here, students understand functions as a rule that assigns to each input exactly one output (8.F.A.1). The main focus is on linear functions and understanding these basic concepts of the linear function:* Linear functions can be represented multiple ways (algebraically, graphically, numerically in tables, or by verbal descriptions).
* The equation y = mx+b is defining a linear function, whose graph is a straight line
* Those relationships that do not give you a line are not linear functions
* Linear functions can be modeled
* The rate of change and initial value of the function can be determined from a description of the relationship or from two (x, y) values
* Qualitative features of a graph (where the function is increasing or decreasing, linear or nonlinear)
 |
| ***The part of the story happening in this unit*** | In this unit, functions are formally defined; students begin to use formal notation and language of functions. Students recognize that the input/output relationship is a correspondence between two sets: the domain and the range. Additionally, they will see several examples of function notation in different contexts to help them develop fluency with the notation. It is important to note that functions are a unifying concept that cuts across much of the high school content. As such the function standards are often supporting standards. This unit puts functions front and center to be the primary object of attention.Another object that is formally defined in this unit is the arithmetic sequence. Students should be familiar with patterns and sequences from previous years but here the idea of describing these sequences as functions is new.Building off the idea of a rate of change and slope from 8th grade, students now examine the behavior of non-linear functions. They describe key aspects of the graph and now calculate and interpret the average rate of change over a specified interval. |
| ***The story after this unit*** | Students will encounter functions for the rest of their math careers and even sometimes in real life!Functions are a unifying theme throughout the rest of high school mathematics and beyond. Students who go on to study calculus will need and use an understanding of functions. Geometric transformations are viewed as functions sending points in the plane to points in the plane. |

**UNIT FLOW SUMMARY**

|  |  |
| --- | --- |
| **UNIT F1** (14 - 15 days) | **Functions**  |
| **Section 0** (1 day) | **Diagnostic Pre-Unit Assessment** |
| **Section 1** (1 day) | **Graphing Stories & Exploring Functions** |
| **Section 2** (1- 2 days) | **Definition of Function, Domain & Range** |
| **Section 3** (.5 - 1 day) | **Interpreting Function Notation in Context** |
| **Section 4** (1 days) | **Arithmetic Sequences** |
| **Section 5** (1 days) | **Mid-Unit Assessment (could be omitted for shorter units)** |
| **Section 6** (3 days) | **Absolute Value, Piecewise & Step Functions** |
| **Section 7** (2 - 3 days) | **Interpret Functions in Context**  |
| **Section 8** (2 days) | **Average Rate of Change** |
| **Section 9** (2-3 days) | **Rational Functions** |
| **Section 10** (2 days) | **Inverse Functions** |
| **Section 11** (1 day) | **Summative Assessment** |

|  |  |
| --- | --- |
| **Section 0:** 1 day | **Diagnostic Pre-assessment** |
| **Pre-Unit Assessment Targets** | Diagnose students’ ability to* Distinguish a function from a non-function, given different representations [8.F.1]
* Distinguish a linear from a nonlinear function, given different representations [8.F.2]
* Construct a linear function given two points [8.F.4]
* Find and interpret the rate of change of a situation [8.F.4]
* Interpret the graph of a function in a context [8.F.5]
 |
| **Sample Assessment Items** |  [*Pre- assessment*](https://docs.google.com/document/d/1xIkZpHTbQx_D85kAO7JI5GAjfJvNHP0EgBJyTmXqybM/edit?usp=sharing) |

|  |  |
| --- | --- |
| **Section 1:** 1 day | **Graphing Stories & Exploring Functions** |
| **Mathematical Goals** | Students will...* Interpret key features of graphs in terms of the quantities and sketch graphs showing key features F.IF.4 F.IF.7
* Understand that a function from one set to another set assigns to each element of the domain exactly one element of the range F.IF.1
 |
| **Narrative overview of section**(and how the standards are achieved) | Over the course of this unit, students will begin their formal study of functions. They will look at function notation, what it means to be a function, how to interpret functions in a given context and analyze them using different representations. The goal of this section is to connect students with graphing functions in a context that is visual and tangible for them to understand.  |
| **Sample Activity 1.1** | [*Graphing Stories*](http://graphingstories.com/), Dan Meyer & Buzz Math**WHAT:** The Graphing Stories website has numerous short videos that capture a change in something (height/width/volume/length) with respect to time for 15-seconds. Students watch the video a few times and then try to graph the given situation.**WHY:** The purpose of beginning with this activity is to provide students with an opportunity to build on their prior knowledge of graphing before beginning the formal discussion about functions (that will appear in section 2). Functions are a way of describing a relationship between quantities in a context. Students must model the mathematics MP.4 being shown while attending to precision MP.6 in order to create a realistic graph of the given context F.IF.7. Here the teacher can help guide students to some observations about these graphs F.IF.4 that help define these functions without the formal language (what are the input/output values? for each time there is only one position/distance/height/etc.). Additionally, this activity provides students with some non-linear examples of graphs to help students begin to broaden their experiences of how quantities can change over time.  |
| **Sample Activity 1.2** | [*It’s a Date*](http://mathalicious.com/lessons/its-a-date)*,* Mathalicious**WHAT:** Because we use it every day, we tend to forget that our calendar is pretty strange. The year is broken up into months of different lengths, seemingly without much of a pattern. The same date can occur on different days of the week in different years. Is this really the best way to carve up a year? In this lesson, students examine some other ways to keep track of dates, and use number sense and function concepts to convert among different calendars. What might a more reasonable calendar look like?**WHY:** In an engaging context, students can discuss the ideas behind what makes a rule a function F.IF.1 and why some functions have inverses and others don’t. These could be treated and written formally, or discussed broadly and informally, depending on the teacher’s goals MP.3. This lesson also includes the idea of function composition: while it’s straightforward to apply y = f(x) and then g(y), it would be difficult to write a concise rule that does both -- it’s simpler to state it as g(f(x)). As with the other ideas in this lesson, the composition could be treated informally, or written with formal notation if a teacher wanted to address F.BF.1.c more explicitly. |
| **Focus Standards** | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ |
| **Mathematical Practices** | MP.3, MP.4, MP.6  |

|  |  |
| --- | --- |
| **Section 2:** 1 - 2 days | **Definition of Function, Domain, Range** |
| **Mathematical Goals** | Students will...* Understand domain & range and, specifically, that a function assigns to each element in the domain exactly one element of the range F.IF.1
* Understand function notation F.IF.2
 |
| **Narrative overview of section**(and how the standards are achieved) | In this section, students understand the formal definition of a function and how it relates to the domain and range. Additionally, students are introduced to function notation and they begin to practice using this notation in terms of a context. Students have worked with functions in the past but this is the first time that the definition and notation are explicitly examined. |
| **Sample Activity 2.1** | [*Interpreting the Graph*](http://www.illustrativemathematics.org/illustrations/636), Illustrative Mathematics**WHAT:** This task is designed to help students learn to read information about a function from its graph, by asking them to show the part of the graph that exhibits certain properties F.IF.1. **WHY:** The purpose of this activity is to provide an opportunity for students to connect function notation with the corresponding graphical context. By connecting the function notation to the prior knowledge of graphing on a cartesian plane and seeing several examples of function notation in use, students develop their understanding of the structure of function notation MP.7. The graphs are represented as f(x) and g(x) and all given conditions are given in function notation; thus this task provides an opportunity for students to begin working with and understanding function notation F.IF.2. |
| **Sample Activity 2.2** | [*Using Function Notation* I](http://www.illustrativemathematics.org/illustrations/598), Illustrative Mathematics**WHAT:** This task asks whether it’s valid to simplify C(x) = 1.25x + 2500 by dividing both sides by x and writing C = (1.25x + 2500)/x. **WHY:** As students begin to understand and use function notation, this problem provides students an opportunity to grapple with a very common misconception seen when using function notation F.IF.1. |
| **Sample Activity 2.3** | [*The Customers*](http://www.illustrativemathematics.org/illustrations/624)*,* Illustrative Mathematics**WHAT:** This task extends the idea of a function to a non-algebraic context. It helps to reinforce the idea of what makes a function a function and what it looks like when something is not a function.**WHY:** One of the main ideas for students to understand in this particular section is that for each input (element in the domain) there is exactly one output (element in the range) F.IF.1. This is achieved by having two examples for students to compare and contrast why one of the rules is a function while the other is not, students begin to notice the structures necessary for a function to be a function MP.7. Taking the idea of a function and presenting it in a non-algebraic example also helps students to see the idea of functions in a real-world context. |
| **Sample Activity 2.4** | [*Points on a Graph*](http://www.illustrativemathematics.org/illustrations/630)*,* Illustrative Mathematics**WHAT:** This task looks at function notation and helps students to see the placement of the independent and dependent variables in function notation.**WHY:** This activity necessitates students to make sense of the meaning of function notation and the graph of a function as ordered pairs (x, f(x)) F.IF.1 . A rationale for the task is that it is abstract and has no context, and hence the students must makes sense of the notation itself MP.8. |
| **Focus Standards** | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x)denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=f(x).  F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| **Mathematical Practices** | MP.7, MP.8 |

|  |  |
| --- | --- |
| **Section 3:** 1 day | **Interpreting Function Notation in Context** |
| **Mathematical Goals** | Students will...* Understand function notation in various contexts F.IF.2
* Interpret statements that use function notation in various contexts F.IF.2
 |
| **Narrative overview of section**(and how the standards are achieved) | In this section students apply their understanding of function notation in various contexts. Students interpret and describe statements given in function notation and understand how these connect to the given context.  |
| **Sample Activity 3.1** | [*Cell Phones*,](http://www.illustrativemathematics.org/illustrations/634) Illustrative Mathematics**WHAT:** This task provides students with a context and four statements about the context given in function notation. The four parts of the task provide a logical progression of exercises for advancing understanding of function notation and how to interpret it in terms of a given context.**WHY:** The purpose of this activity is to have an opportunity for students to explain, in their own words, how they understand the notation F.IF.2 starting with a concrete example (f(10)=100.3) and leading to a more abstract example (n = f(t)) while staying within a given context MP.2. |
| **Sample Activity 3.2** | [*Yam in the Oven*](http://www.illustrativemathematics.org/illustrations/625)*,* Illustrative Mathematics**WHAT:** This task explores how to interpret function notation. **WHY:** The purpose of this activity is to have an opportunity for students to explain, in their own words, how they understand function notation F.IF.2. Additionally, students must demonstrate that they understand the notation enough to be able to compare values in function notation (f(5) < f(10)) in the given context. |
| **Target Standard** | F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. |
| **Mathematical Practices** | MP.2 |

|  |  |
| --- | --- |
| **Section 4:** 1 days | **Arithmetic Sequences Using Function Notation** |
| **Mathematical Goals** | Students will...* Recognize that sequences are functions F.IF.3
* Recognize that sequences can sometimes be defined recursively F.IF.3
* The domain of a sequence is a subset of integers F.IF.3
 |
| **Narrative overview of section**(and how the standards are achieved) | Students have been looking at and exploring patterns throughout their mathematical careers. In this section, students identify that sequences (patterns) are functions. Further, for those sequences that can be defined recursively, students learn how to write and notate accordingly.  |
| **Sample Activity 4.1** | [*Snake on a Plane*](https://www.illustrativemathematics.org/illustrations/1695)*,* Illustrative Mathematics **WHAT:** This task presents students with a game, Snake, that they may have seen before. Students explore how the snake grows, looking at the function both recursively and algebraically F.IF.3. There is a visual that shows how the snake grows (<http://i.imgur.com/dAtcCfH.gif>) to help students visualize what is happening in this context. **WHY:** The approach of this problem where students first look at the sequence (pattern) then work to define it both recursively and algebraically helps students to recognize and define sequences as functions MP.7.  |
| **Sample Activity 4.2** | [*Romeo and Juliet*](http://mathalicious.com/lessons/romeo-juliet)*, Mathalicious***WHAT**: When two people date, how one person feels often depends on how the other person feels. Your feelings are also informed by your own feelings, not just your partner's. In a certain well-known tragedy by Shakespeare, this feedback loop got way out of control. In this lesson, students investigate the effect of coefficients on recursive functions. They first analyze a simpler version, where Romeo's feelings each week are simply scaled by Juliet's feelings the previous week. Then, they analyze a more complicated rule, where Romeo's feelings this week depend on both Juliet's and his own feelings the previous week. Can romantic love be modeled with mathematics? We're about to find out!**WHY**: The notation, and even the basic idea behind, recursively defined functions can be difficult for students to grasp. The purpose of this lesson is to put recursively-defined functions in an engaging context to help students make sense of how they work and the notation used to describe them F-IF.3. This lesson is an opportunity for students to reason abstractly MP.2 about something that’s hard to measure -- someone’s feelings! |
| **Sample Activity 4.3** | [*Visual Patterns*](http://www.visualpatterns.org/)*,* Fawn Nguyen**WHAT:** Also called “pile problems,” students are presented with a sequence of physical objects like blocks or toothpicks. They are prompted to sketch the next item in the sequence, predict the number of objects in an item far down the road, and generate an equation relating the item number to the number of objects F.IF.3. For arithmetic sequences, suggest starting with pattern numbers 2, 4, 7, 9, 10, 14, 15, 17, 18. [The form provided](http://www.visualpatterns.org/#!teachers/component_14113) could easily be adapted for developing the idea of sequences by asking students to write the relationship as a function, and also also generate a recursive definition of the function. **WHY:** The concrete objects and their arrangement in space give students something to grab onto to support their reasoning. Generating the functions can be approached through multiple entry points: making and reasoning about a table, plotting points representing (item number, number of objects), or a verbal description of how the number of objects is changing. Since students are familiar with writing the equations of lines from eighth grade (and possibly unit A1, if that happened before this unit), they can use what they already know about constant rate of change and writing the equations of lines to help them understand arithmetic sequences MP.8. The teacher will have to do further work defining “sequence” and “arithmetic sequence”, but these concrete examples can provide a context for those definitions. |
| **Target Standards** | F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0)=f(1)=1, f(n+1)=f(n)+f(n−1) for n≥1. |
| **Mathematical Practices** | MP.7, MP.8 |

|  |  |
| --- | --- |
| **Section 5:** 1 day | **Diagnostic Mid-assessment** |
| **Pre-Unit Assessment Targets** | Assess students’ ability to* interpret function notation in terms of a graph and the relationship between the domain and range [F.IF.1]
* express points on a graph, and features of a graph, using function notation [F.IF.1]
* understand function notation and how to interpret statements in function notation in terms of a context [F.IF.2]
* interpret an arithmetic sequence defined recursively and/or defined with a list [F.IF.3]
 |
| **Sample Activity** | [*Mid-assessment*](https://docs.google.com/document/d/1eYd414Yd_4sHlTmVJO6DlyNEoZlV9EO7rDEUga3dLpM/edit?usp=sharing) |

|  |  |
| --- | --- |
| **Section 6:** 3 days | **Absolute Value, Piecewise, Step Functions** |
| **Mathematical Goals** | Students will...* Recognize and understand piecewise functions, step functions and absolute value functions F.IF.7b
* Understand the above functions expressed symbolically and can show key features of the graphs F.IF.7
 |
| **Narrative overview of section**(and how the standards are achieved) | In this section students are introduced to graphs that they may not have seen before. Students understand that not all contexts can be represented linearly and that sometimes other types of functions are better used to address these situations. Additionally, students develop a strong understanding of connecting a graphic representation to a given situation, and through examination of the given data, students develop an understanding of how to properly graph the situation in a given context. |
| **Sample Activity 6.1** | [*The Parking Lot,*](http://www.illustrativemathematics.org/illustrations/588) Illustrative Mathematics**WHAT:** This problem provides a real-world context of a piecewise step function. Students must graph the cost of parking in a specific parking garage where the cost increases each half-hour or fraction thereof F.IF.7.**WHY:** The purpose of this activity is to provide an opportunity for students to examine a piecewise step function in a real-world context F.IF.7.b. The idea that the price “jumps” $.50 each half-hour or fraction there-of gives rise to why a single linear function is not an appropriate model for this problem MP.4. This task lends itself to a great discussion about domain and range F.IF.1 and why open and closed points are necessary in order for the graph to be a function. |
| **Sample Activity 6.2** | [*Bank Account Balance*](https://www.illustrativemathematics.org/illustrations/1833)*,* Illustrative Mathematics**WHAT:** This problem presents the students with a graph that shows the balance in a bank account over a seven-day period. The graph is shown as a continuous function but because the actual function (bank account balance over a time period) would vary discretely, students must correctly make a step function to represent the daily balance.**WHY:** This task provides students with an opportunity to explain why the given function model is not the most appropriate model for the given context, specifically comparing a continuous function with a step function MP.2. Part b. of this task is open-ended and allows for various student models MP.4 to best represent the bank account balance (step function F.IF.7.b and bar graph would be two possible models). This could lend itself to a class discussion where students must support and explain why their model best represents the given situation MP.3.  |
| **Sample Activity 6.3** | [*Graphing Absolute Value of Guessing Error*](http://function-of-time.blogspot.com/2010/06/absolute-value-both-rigorous-and-in.html)*, f(t)***WHAT:** The teacher prepares a sealed jar of candy and records many participants’ guesses about the number of candies in the jar. Students are provided with a spreadsheet of everyone’s guesses. In the process of answering their own questions about the data (for example, “who made the best guess? the worst? how many guessers did better than average?” students are prompted to create and interpret a scatterplot of guess vs distance from correct number of candies, resulting in the graph of an absolute value function MP.4. **WHY:** The “distance from a value” rooted in a concrete setting is a meaningful way to approach graphs of absolute value equations F.IF.7.b. |
| **Target Standards** | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).F.IF.7.b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. |
| **Mathematical Practices** | MP.2, MP.3, MP.4 |

|  |  |
| --- | --- |
| **Section 7:** 2-3 days | **Interpret Functions in Context**  |
| **Mathematical Goals** | Students will...* Interpret key features of graphs and tables in terms of the quantities presented F.IF.4
* Sketch graphs showing key features F.IF.4
* Relate the domain of a function to its graph F.IF.5
* Relate the domain to the quantitative relationship it describes F.IF.5
 |
| **Narrative overview of section**(and how the standards are achieved) | In this section, students really begin to understand functions and connect them to various contexts. This includes understanding how to read and interpret function models presented both graphically and in table form. Students understanding of appropriate values for the domain and range are also refined in this section where students must analyze all possible input/output values in the given contexts. There are several tasks that explore the mathematical goals for this section. One way to approach this section would be with a jigsaw activity. In small groups, students become experts on one particular problem. Students are then regrouped with new peers to teach/explain the meaning of and how to solve their problem [MP.3].  |
| **Sample Activity 7.1** | [*Oakland Coliseum*](http://www.illustrativemathematics.org/illustrations/631)*,* Illustrative Mathematics**WHAT:** In this problem students are given information about seating capacity and cost of tickets at the Oakland Coliseum. The revenue for the Raider’s organization is a function of the number of people in attendance. Students are asked to find the domain and range of this function.**WHY:** This problem presents a real-world context for domain and range. Students demonstrate that they understand the idea of domain as all possible input values and the range as all possible output values. In the context of this problem, they must be careful to not include negative and non-integer values for both the domain and range F.IF.5.  |
| **Sample Activity 7.2** | [*Warming and Cooling,*](http://www.illustrativemathematics.org/illustrations/639)Illustrative Mathematics**WHAT:** This task provides students with a graph showing temperature change over a given time period. Students analyze certain key features of the graph (maximum, decreasing) in the given context.**WHY:** This straightforward graphing task allows students to develop their understanding of how to connect function notation with a corresponding graph in a given context. Students initially began to connect function notation to graphing in section one of this unit; here students build on that to develop their understanding of key features of a graph (maximum, decreasing) and their significance in a real-world context F.IF.4 MP.7. |
| **Sample Activity 7.3** | [*Influenza epidemic*](http://www.illustrativemathematics.org/illustrations/637)*,* Illustrative Mathematics**WHAT:** In this problem, students are presented with a graph that represents an influenza epidemic as it spreads across a city. Students must then interpret and explain different statements about the graph, all presented in function notation.**WHY:** As explained in the commentary for this problem, “The principal purpose of this task is to probe students’ ability to correlate symbolic statements about a function using function notation with a graph of the function, and to interpret their answers in terms of the quantities between which the function describes a relationship.”F.IF.4 Part d) of this problem necessitates students to be able to comfortably switch between function notation and an expression for a function F.IF.2 MP.7. Additionally, as no exact numerical data can be extracted from this graph, students must attend to precision MP.6 when interpreting and explaining the given statements.  |
| **Sample Activity 7.4** | [*How is the Weather?*](http://www.illustrativemathematics.org/illustrations/649)[*,*](http://www.illustrativemathematics.org/illustrations/639)Illustrative Mathematics**WHAT:** In this task, students are given three graphs and three statements, each statement being represented by one of the graphs. Students must read and interpret the graphs in order to properly connect the graph to the correct statement.**WHY:** The purpose of this problem is for students to identify key features of the given graphs (increasing, decreasing, maximum, end behavior) and once identified, connect these to the given statements F.IF.4. At first glance the first and third graphs look very similar, students must demonstrate careful attention to detail MP.6 to point out that the scale of the vertical axis is different in these two graphs making their stories unique. |
| **Sample Activity 7.5** | [*Telling a Story with Graphs*](http://www.illustrativemathematics.org/illustrations/650)[*,*](http://www.illustrativemathematics.org/illustrations/639)Illustrative Mathematics**WHAT:** Three graphs are presented, each showing a measurable weather-related quantity over a time period. Students are asked to tell a story that is first represented by each graph independently and then a more detailed story that relates the graphs to each other.**WHY:** The purpose of this problem is to really help students see beyond function graphs as simply a connection of points. In order to write a story that explains what the graph is showing students must demonstrate that they understand how the variables are changing with respect to each other. They must also understand key features of the graph (increasing/decreasing/maximum/minimum) F.IF.4 to add depth and detail to their story. Further analysis is required to create a story that connects the graphs to each other as students must not only understand each graph but how that data is related to what is shown in the other graphs MP.4 MP.6. |
| **Target Standards** | F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★ |
| **Mathematical Practices** | MP.4, MP.6, MP.7  |

|  |  |
| --- | --- |
| **Section 8:** 2 days | **Average Rate of Change** |
| **Mathematical Goals** | Students will...* Calculate the average rate of change of a function over a specified interval F.IF.6
* Interpret the average rate of change of a function over a specified interval F.IF.6
* Estimate the rate of change from a graph F.IF.6
 |
| **Narrative overview of section**(and how the standards are achieved) | In grade 8, students learned that rate of change of a linear function is its slope. And because the slope of a line is constant, “the rate of change” has an unambiguous meaning for a linear function (from the [Progressions Document](http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf), page 9). In this section, students are introduced to rates of change that are not constant as they apply the idea to nonlinear functions. Students examine and interpret multiple models of nonlinear functions to best understand the idea of average rate of change of a function over a specified interval. |
| **Sample Activity 8.1** | [*Temperature Change,*](http://www.illustrativemathematics.org/illustrations/1500)Illustrative Mathematics**WHAT:** In this task, students are given a basic table with temperature and time. They must decide between the two given intervals, over what interval the temperature decreased the fastest. **WHY:** As students begin to understand the concept of average rate of change, this problem provides students an opportunity to grapple with a common misconception seen with rate of change; average rate of change vs. absolute change. By having students explain their reasoning and how they interpreted the greatest decrease in temperature from the given data MP.3, students gain a concrete understanding of the importance of having things in the same units in order to compare the data in a more meaningful way. For this particular task, you must look at the average rate of change F.IF.6 in order to have the quantities in equivalent units. |
| **Sample Activity 8.2** | [*The High School Gym*](http://www.illustrativemathematics.org/illustrations/577)*,* Illustrative Mathematics**WHAT:** In this task, students are given a general story about the temperature in a high school gym over a period of time. In the first two parts of this task, students identify the proper function with correct notation. In the second two parts, students interpret and examine the given expressions involving average rate of change. **WHY:** The first two parts of this task allow students to demonstrate their understanding of what a function is and how to interpret function notation F.IF.1. The second two parts of this task allow students to demonstrate, not only how to calculate an average rate of change but also how to identify an average rate of change in function notation F.IF.6 MP.7. |
| **Sample Activity 8.3** | [*Mathemafish Population,*](http://www.illustrativemathematics.org/illustrations/686)Illustrative Mathematics**WHAT:** In this task, students are presented with data both in table form and graphically. Students must read and interpret this data in the given context in order to construct a summary report that details the average rates of change from the data along with the meaning of this data in the given context.**WHY:** This task provides students an opportunity to use average rate of change calculations F.IF.6 to construct a viable argument MP.3 in the given context. Students demonstrate an understanding of the meaning/significance of the average rate of change values by comparing and contrasting rates over various intervals. Additionally, these rates, when compared with the given graph, allow students an opportunity to connect key features F.IF.4 of the graph to their mathematical calculations. |
| **Target Standards** | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x)denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=f(x). F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ |
| **Mathematical Practices** | MP.3, MP.7 |

|  |  |
| --- | --- |
| **Section 9:** 2-3 days | **Rational Functions** |
| **Mathematical Goals** | Students will...* Build a function that models a rational relationship between two quantities F.BF.1
* Interpret functions that arise in applications in terms of a context F.IF.B
* Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. F.IF.5
* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. F.IF.4
* Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. F.IF.7.d
 |
| **Narrative overview of section**(and how the standards are achieved) | In this section, students encounter another basic kind of non-linear function of the form 1/x. Starting from concrete relationships, students are presented with situations that result in quantities varying inversely and their asymptotic behavior. In the process, they will sketch graphs showing key features given a verbal description of the relationship. Conversely, they’ll interpret key features of graphs and tables representing rational functions in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; symmetries; and end behavior. |
| **Sample Activity 9.1** | [The Canoe Trip, Variation 1,](http://www.illustrativemathematics.org/illustrations/386) Illustrative Mathematics**WHAT:** The Canoe Trip, Variation 1, is a task where students find an equation that models how long a trip upstream will take, based on the speed of the current. Students interpret intercepts and vertical asymptotes of the graph based on the context of the problem.**WHY:** This task was selected because (1) it is an authentic and easy to understand real-world context that can be modeled by a rational function, (2) the functions derived do not require an excessive amount of algebraic manipulation to allow students to better focus on the structure and behavior of the model and (3) the asymptotes have a useful meaning within the context of the problem. F.IF.4 Although this task doesn’t incorporate the entire modeling cycle, students engage in aspects of MP.4 because they create a function to represent a given situation, and then interpret features of its graph in a context. |
| **Sample Activity 9.2** | [*Average Cost*](http://www.illustrativemathematics.org/illustrations/387)*,* Illustrative Mathematics**WHAT:** In this task, students analyze the average cost of a DVD as the total amount of DVDs produced increases. Students are given the cost function for producing DVDs and after analyzing how the average cost decreases as you produce more, they must create the average cost function and find the domain of this function F.IF.5. Students analyze the information given in this problem through function notation, a table with values and graphically.**WHY:** In this task, students have an opportunity to build their understanding of average rate of change functions through a concrete example. Students develop this understanding by first creating a table of values and using this data to construct the average rate function. Since they perform a number of calculations and then generalize the operations with an equation, they are looking for and expressing regularity in repeated reasoning MP.8. By graphing this function in the last step of this problem, students draw connections between the table of values, the average rate of change function and the corresponding graph. The problem gives a nice context for understanding end behavior of a function that approaches a horizontal asymptote. F.IF.4 |
| **Sample Activity 9.3** | [Graphing Rational Functions](http://www.illustrativemathematics.org/illustrations/1694), Illustrative Mathematics**WHAT:** Graphing Rational Functions is a task where students use sliders in Desmos to make connections between the equations of rational functions and their asymptotes. This task starts with an exploration of the graphs of two functions whose expressions look very similar but whose graphs behave completely differently. At first glance this is surprising but can be explained by a closer look at the functions' expressions.**WHY:** This is a nice task to follow the previous two tasks, both of which had a context provided to strengthen the conceptual understanding on an asymptote. This task now abstracts the idea of an asymptote, where students are trying to see a pattern between given equations and graphs. Specifically, it illustrates how a function’s equation is connected to the location of the vertical asymptotes F.IF.7.d. The important point of the task is to connect features of the graph with corresponding features or properties of the formulas MP.7. |
| **Sample Activity 9.4** | [Carpe Donut](http://mathalicious.com/lessons/donut-stand), Mathalicious**WHAT**: Carpe Donut in Charlottesville, Virginia, has an interesting pricing scheme. You can buy one donut for $2, two donuts for $3, three for…well, you get the idea. This means that two people could pay less by purchasing their donuts together. Three people could do even better. So how does the average cost per donut change, and how much should we be paying for each? In this lesson, students use linear, rational, and piecewise functions to describe the total and average cost of an order at Carpe Donut. Through this, they’ll see the benefits of buying in bulk, and will find a least-expensive way to purchase these delicious donuts.**WHY**: Carpe Donut is about creating an accurate model for the pricing scheme of a donut shop. At this stage of the unit, it provides a context for bringing several ideas together, including building, graphing, F.IF.7 and interpreting rational F.IF.7.d and piecewise F.IF.7.b functions and relating the domain of a function to its graph F.IF.5. Because students are presented with a real-life situation and must construct and interpret a mathematical model they engage in MP.4. By performing several computations and then generalizing with an equation, students are expressing regularity in repeated reasoning MP.8. Act Two of this lesson should be considered optional in this unit, as it is suited for classes or students who are up for a challenge. By complicating the model with “the thirteenth donut is free,” it gets into modular arithmetic and generating a limit argument for the average cost of a large number of donuts. This would be an opportunity for students to use graphing software strategically MP.5 |
| **Focus Standards** | F.IF.4, F.IF.5, F.IF.7, FIF.7b, F.IF.7d |
| **Mathematical Practices** | MP.4, MP.5, MP.7, MP.8 |

|  |  |
| --- | --- |
| **Section 10:** 2 days | **Inverse Functions** |
| **Mathematical Goals** | Students will...* Find the inverse function of basic functions F.BF.4
* Write an expression for the inverse F.BF.4a
 |
| **Narrative overview of section**(and how the standards are achieved) | In this unit, students learn about the basics of inverse functions. Through a real-world context, students learn the importance of inverse functions and how to express them in proper notation. |
| **Sample Activity 10.1** | [*Decoder Ring*](http://mathalicious.com/lessons/decoder-ring)*,* Mathalicious**WHAT:** Through the lens of cryptography, learning how to both encrypt and decode messages, students learn about functions and their inverses. Students look at both algebraic and graphic ciphers to help them encrypt messages. Once familiar with how ciphers work, students then work to encrypt messages. In the end, students look at what ciphers are effective and which are not in the encryption process. **WHY:** The purpose of this task is for students to understand what inverse functions are and why they are necessary in a real-world contextF.BF.4 F.BF.4a. Through reading, interpreting and using ciphers, students demonstrate an understanding of functions F.IF.1. Students must then persevere MP.1 in their mathematical reasoning MP.2 to identify and understand what is special about the certain functions (they have inverses) that make effective encryption rules. |
| **Sample Activity 10.2** | [*Your Father*](http://www.illustrativemathematics.org/illustrations/589)[,](http://www.illustrativemathematics.org/illustrations/634) Illustrative Mathematics**WHAT:** This task gives a basic function rule set in a non-algebraic context and students must initially identify why the rule is a function. In the second part of the task, students look at what must be true for the rule in part one to have an inverse. Lastly, the third part of the problem connects the domain of a function to its inverse.**WHY:** The purpose of this task is to give students an opportunity to understand what is necessary for a function to have an inverseF.BF.4 F.BF.4a. Students must first demonstrate their understanding of what makes something a function F.IF.1 before they extend this idea to the inverse function (seen in part a. of this problem). The non-algebraic context of this task allows students to interpret, explain and reason with the idea of an inverse function in a setting that is familiar to them before the algebraic demands of an inverse and its notation are introduced.  |
| **Sample Activity 10.3** | [*Temperatures in degrees Fahrenheit and Celsius*](https://www.illustrativemathematics.org/illustrations/501)*, Illustrative Mathematics***WHAT:** In the first part of this task, students construct a linear function given two temperature data points (input-output pairs) representing the relationship between temperature on the Fahrenheit and Celsius scale F.LE.2. Students then use the constructed function from part one to find the inverse functionF.BF.4 F.BF.4a. The third part of the task requires quantitative reasoning MP.2 when students are asked if there is a temperature which is the same in Fahrenheit and Celsius.**WHY:** The purpose of this task is to provide an opportunity for students to understand and construct inverse functions in a real-world context.  |
| **Sample Activity 10.4** | [*US Households*](https://www.illustrativemathematics.org/illustrations/234)*, Illustrative Mathematics*WHAT: In this task, students are given a table of information relating the number of households in the US in certain years. Using this data, students construct a linear function F.LE.2 that models the number of households as a function of the year MP.4. In the last parts of the task, students find the inverse functionF.BF.4 F.BF.4a and then use this to interpret a given value of the inverse function. **WHY:**  The purpose of this task is to provide an opportunity for students to understand and construct inverse functions in a real-world context. Additionally, this task presents inverses using inverse notation which students may not have seen at this time so this task can also serve as an introduction to inverse notation.  |
| **Focus Standards** | F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x)denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y=f(x). F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).F.BF.4F.BF.4a Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x ≠ 1. |
| **Mathematical Practices** | MP.1, MP.2, MP.4 |

|  |  |
| --- | --- |
| **Section 11:** 1 day | **Summative Assessment** |
| **Summative Assessment Targets** | * Students can identify the domain and range when given a function [F.IF.1]
* Students can create a step function when given data [F.IF.7.b]
* Students can interpret statements that use function notation in a context [F.IF.2]
* Students can find the next values in a recursive sequence and when given a sequence can identify if it is a function [F.IF.3]
* Students can identify key features of graphs and can interpret the meaning of function notation in a given graph [F.IF.4]
* Students can identify and relate the domain and range of a function in a given context [F.IF.5]
* Students can calculate the average rate of change of a function over a specified interval [F.IF.6]
 |
| **Sample Activity** | [Link to Assessment](https://docs.google.com/document/d/1tyfKh3kY_tp7Px5l5hAw0vZIA5dwr_AwQHT6esdbjb4/edit?usp=sharing) |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1.1 | 1.2 | 2.1 | 2.2 | 2.3 | 2.4 | 3.1 | 3.2 | 4.1 | 4.2 | 4.3 | 6.1 | 6.2 | 6.3 | 7.1 | 7.2 | 7.3 | 7.4 | 7.5 | 8.1 | 8.2 | 8.3 | 9.1 | 9.2 | 9.3 | 9.4 | 10.1 | 10.2 | 10.3 | 10.4 |
| F.IF.1 |  | **x** | **x** | **x** | **x** | **x** |  |  |  |  |  | **x** |  |  |  |  |  |  |  |  | **x** |  |  |  |  |  | **x** | **x** |  |  |
| F.IF.2 |  |  | **x** |  |  |  | **x** | **x** |  |  |  |  |  |  |  |  | **x** |  |  |  |  |  |  |  |  |  |  |  |  |  |
| F.IF.3 |  |  |  |  |  |  |  |  | **x** | **x** | **x** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| F.IF.4 | **x** |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** | **x** | **x** |  |  | **x** | **x** | **x** |  |  |  |  |  |  |
| F.IF.5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** |  |  |  |  |  |  |  |  | **x** |  | **x** |  |  |  |  |
| F.IF.6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** | **x** |  |  |  |  |  |  |  |  |
| F.IF.7 | **x** |  |  |  |  |  |  |  |  |  |  | **x** |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** |  |  |  |  |
| F.IF.7b |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** | **x** |  |  |  |  |  |  |  |  |  |  |  | **x** |  |  |  |  |
| F.IF.7d |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** |  |  |  |  |
| F.BF.4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** | **x** | **x** |
| F.BF.4a |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** | **x** | **x** |
| F.LE.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **x** | **x** |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1.1 | 1.2 | 2.1 | 2.2 | 2.3 | 2.4 | 3.1 | 3.2 | 4.1 | 4.2 | 4.3 | 6.1 | 6.2 | 6.3 | 7.1 | 7.2 | 7.3 | 7.4 | 7.5 | 8.1 | 8.2 | 8.3 | 9.1 | 9.2 | 9.3 | 9.4 | 10.1 | 10.2 | 10.3 | 10.4 |
| MP.1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | x |  |  |  |
| MP.2 |  |  |  |  |  |  | x |  |  | x |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  |  | x |  | x |  |
| MP.3 |  | x |  |  |  |  |  |  |  |  |  |  | x |  |  |  |  |  |  | x |  | x |  |  |  |  |  |  |  |  |
| MP.4 | x |  |  |  |  |  |  |  |  |  |  | x | x | x |  |  |  |  | x |  |  |  | x |  |  | x |  |  |  | x |
| MP.5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | x |  |  |  |  |
| MP.6 | x |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | x | x | x |  |  |  |  |  |  |  |  |  |  |  |
| MP.7 |  |  | x |  | x |  |  |  | x |  |  |  |  |  |  | x | x |  |  |  | x |  |  |  | x |  |  |  |  |  |
| MP.8 |  |  |  |  |  | x |  |  |  |  | x |  |  |  |  |  |  |  |  |  |  |  |  | x |  | x |  |  |  |  |