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| **Unit** | **A2** |
| **Title** | Exponential Functions 1 |
| **Target Standards** | **F.LE.1, F.LE.1a, F.LE.1c, F.LE.2, F.LE.3, F.LE.5, F.IF.7e, F.BF.1a, A.CED.1, F.IF.8b** |
| **Mathematical Goals** | Students will…   * Understand the basic notation and graph of an exponential function F.LE.5 * See different representations of real-world exponential models F.LE.1 * Distinguish between situations that can be modeled with linear and exponential functions F.LE.1 * Create tables and graphs to describe exponential models F.IF.7e * Notice and point out linear and exponential functions when provided with data in a table, graph, description or function F.LE.2 * Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly F.LE.3 * Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals F.LE.1a * Be able to apply exponential functions in both growth and decay situations F.LE.1c * Write the appropriate exponential function f(t) = a(1±r)^t, when given a starting value and and rate of change over time F.LE.2 * Identify the rate of increase or decrease given an exponential function F.LE.2 * Recognize that exponential functions have a constant percent rate per unit interval F.LE.1c * Write an exponential function in a real-world context F.BF.1a |
| ***The story before this unit (including prior knowledge)*** | In the 9th grade, students should be very familiar with linear functions from the F1 unit. They have been formally introduced to functions and function notation and have explored the behaviors and traits of both linear and non-linear functions. Additionally, students have spent significant time graphing, interpreting graphs and have explored how to compare the graphs of two linear functions to each other. |
| ***The part of the story happening in this unit*** | In this unit, students are introduced to exponential functions. Students recognize exponential relationships when data is presented in tables, graphs and real-world contexts. Students compare linear and exponential function models and are able to distinguish between situations that should be modeled with a linear function vs. an exponential function. Students know the basic form of an exponential function (f(x) = a\*bx) and given data points, can create an exponential function model. Students also understand that given time, an exponential model will eventually exceed linear models. |
| ***The story after this unit*** | In A3, students will be introduced to quadratic functions. Students will focus on the basic nature of quadratic functions, contexts in the real world that change quadratically, and the different forms of a quadratic equation and how each one describes something about the graph. Students will also extend their understanding of the exponential model and how it relates to a quadratic model; understanding that an exponential function will eventually exceed both linear and quadratic models.  In the A7 unit, students will focus on a more in-depth understanding of exponential and log functions. |

**UNIT FLOW SUMMARY**

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| **UNIT A2** (12 - 15 days) | **Exponential Functions 1** |
| **Section 0** (1 day) | **Diagnostic Pre-Unit Assessment** |
| **Section 1** (1 day) | **What is Exponential Growth?** |
| **Section 2** (3 - 4 days) | **Understanding Exponential Functions** |
| **Section 3** (3 - 4 days) | **Exponential vs. Linear Growth** |
| **Section 4** (2 - 3 days) | **Percent Growth and Decay** |
| **Section 5** (1 day) | **Bringing It All Together** |
| **Section 6** (1 day) | **Summative Assessment** |

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| **Section 0:** 1 day | **Diagnostic Pre-assessment** |
| **Pre-Unit Assessment Targets** | Diagnose students’ ability to   * manipulate exponents in algebraic expressions [8.EE.A.1] * calculate percent increase/decrease [6.RP.A.3.C] * Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form [7.EE.B.3] * describe a linear relationship [8.EE.C.7] * describe a non-linear (exponential) relationship [8.F.B.5] * compare different data sets over time and describe the differences [8.F.A.2] |
| **Sample Activity** | * [*Link to assessment*](https://docs.google.com/a/hightechhigh.org/document/d/1Qf5SzHUK9hcV8hSinOYEVvIEPaYzDdmxmgG8mFdGk1w/edit) |



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| **Section 1:** 1 day | **What is Exponential Growth?** |
| **Mathematical Goal** | Students will…   * Be introduced to exponential functions and will begin to see the differences between linear and exponential functions F.LE.1 |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, students will first be exposed to exponential functions. As much of their mathematically history has been spent with linear functions, students will see exponential models, but may initially only recognize them as non-linear. Students will build their understanding of the exponential model in an organic way through the modeling process. |
| **Sample Activity 1.1** | [*Paper Folding*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CscEEySjRaSmxQT0E/edit), High Tech High, Jade White  **WHAT:** Students take a regular piece of paper and use it to notice a pattern between the number of folds and the number of regions created, modeling exponential growth MP.4. Students record this data then graph it and use the information to figure out the function (f(x) = 2x) that represents this data. To further model exponential growth, students are asked to figure out how many folds would be necessary to get the height of the paper to reach the moon (hypothetically, of course!). Assuming that the width of the paper is .01mm, the function to represent the paper width is f(x) = .01\*2x, so students are introduced to the basic exponential function f(x) = a\*bx through this activity F.LE.2.  [Mythbusters](http://www.mythbusterstheexhibition.com/try-this-at-home/try-this-entry/) did a short clip on how many times you can fold a piece of paper in half using a paper the size of a football field and for a wrap-up, this [quick video](http://ed.ted.com/lessons/how-folding-paper-can-get-you-to-the-moon) shows the paper folding to the moon idea.  **WHY:** The purpose of this activity is to provide students with a tangible way to understand that a doubling pattern is exponential growth F.LE.1 MP.7. Students should recognize that the growth is non-linear and, while they may not immediately know the growth is exponential, they begin to understand that there are other function families besides linear functions. |
| **Target Standards** | F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.  F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). |
| **Mathematical Practices** | MP.4, MP.7 |

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| **Section 2:** 3 - 4 days | **Understanding Exponential Functions** |
| **Mathematical Goals** | Students will...   * Understand the basic notation and graph of an exponential function F.LE.5 * See different representations of real-world exponential models F.LE.1 * Distinguish between situations that can be modeled with linear and exponential functions F.LE.1 * Create tables and graphs to describe exponential models F.IF.7e |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, students will explore, create and interpret exponential function graphs. Students will learn the basics of an exponential function graph and will understand the conceptual and graphic differences between exponential decay and growth. Students will create graphs to fit given real world data and will interpret key features of the graph in the given context. |
| **Sample Activity 2.1** | M&M Lab: [Growth](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsWFRoVnY0TmlXdk0/edit), [Decay](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsVHNfa2doQldkQnc/edit) & [Conclusion](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsWkZFbmt0TEliaGM/edit), High Tech High, Jade White  **WHAT:** In these two mini-labs, students use M&Ms to create both exponential growth and decay models. Students initially collect data and create graphs by hand making general observations about the growth/decay of the models MP.4. In the second phase of the activity, students represent their data using technology and build on their initial observations to find an exponential function that best fits their data F.LE.5. Students are encouraged to try both a linear and exponential best fit function to see which one truly fits their data best F.LE.1.  **WHY:** The purpose of this activity is to build students understanding of exponential functions. This is done from initial observations made from data in a table and a hand drawn graph to more formal observations, notation and explanations using a graph generated using technology MP.5. |
| **Sample Activity 2.2** | [Graphing Exponential Functions](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsSzg3bmxGY1Vhc1E/edit), High Tech High, Jade White  **WHAT:** In this exploration, students use graphing technology such as Desmos to understand key features and the basics of an exponential graph F.IF.7e. Students are given the definition of an exponential function and, through graphing, figure out why certain constraints are set on the function.  **WHY:** The purpose of this exploration is to give students the opportunity to use technology to model MP.4 the exponential function and to see and interpret what happens when to the function as the base and coefficient values change. While students could simply be given the definition, here the purpose is for students to understand why the given conditions are required in order to maintain an exponential function MP.6. |
| **Sample Activity 2.3** | [*Billions and Billions*](http://mathalicious.com/lessons/7-billion), Mathalicious  **WHAT:** In this lesson, students will explore how many people the Earth is adding and losing each minute, and use this to build an exponential model for human population growth F.IF.7e. Based on the given data, students must analyze the points and reason that, in this context, an exponential model represents the data better than a linear model F.LE.1. They will use their model to predict what the world population will be in the future, and this extends to some bigger questions about what life will be like with more and more people F.LE.5.  **WHY:** The purpose of this task is to provide students an opportunity to understand exponential function modeling MP.4 in a real-world context. Further, students demonstrate proficiency in understanding an exponential model by creating and supporting their predictions about the population on Earth in future years MP.3. |
| **Focus Standards** | F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.  F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.  F.IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |
| **Mathematical Practices** | MP3, MP4, MP5, MP6 |

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| **Section 3:** 3 - 4 days | **Exponential vs. Linear Growth** |
| **Mathematical Goals** | Students will...   * Observe the differences between linear and exponential functions F.LE.1 * Notice and point out linear and exponential functions when provided with data in a table, graph, description or function F.LE.2 * Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly F.LE.3 * Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals F.LE.1a |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, students will compare linear and exponential models. Students are familiar with linear models and linear growth so students will use this base understanding to develop the notion that exponential models will eventually exceed those that are linear. |
| **Sample Activity 3.1** | [*Having Kittens*](http://map.mathshell.org/materials/download.php?fileid=1204)*,* MARS  **WHAT:** In this activity, students model the possible descendents from one kitten in an 18 month period MP.4. Students are given certain parameters about cats and how quickly they can reproduce so students must make sense of these givens as well as make certain logical assumptions about the reproduction process (how many kittens are male/female in a litter? do they always have 4 - 6 kittens per litter? etc.) MP.1 MP.2. Once students understand the givens and make their assumptions, they must construct a viable argument to explain whether they think that one female kitten can have 2000 descendents in just 18 months MP.3.  **WHY:** This problem has many solutions so the emphasis here is on how well students can model and explain their solution. Depending on what assumptions are made, the cat population could grow linearly or exponentially F.LE.1 so this activity provides an excellent segue into comparing the difference in growth between the two models. |
| **Sample Activity 3.2** | [*Linear or Exponential*](https://www.illustrativemathematics.org/illustrations/629)*, Illustrative Mathematics*  **WHAT:** This task provides 5 real-world situations that can be modeled by a linear or exponential function.  **WHY:** Because of the real-world context, this is a good introductory task to provide students with situations of both exponential and linear models to allow them to begin to decipher the differences in the two models F.LE.1. |
| **Sample Activity 3.3** | [*Exponential Growth Versus Linear Growth I*](https://www.illustrativemathematics.org/illustrations/366)*,* Illustrative Mathematics  **WHAT:** This task provides students with two payment options for completing the chore of raking leaves; one is an exponential model and one is linear F.LE.1.  **WHY:** The purpose of this task is for students to calculate and observe how quickly the exponential model surpasses the linear model F.LE.3. |
| **Sample Activity 3.4** | [*Exponential Growth Versus Linear Growth II*](https://www.illustrativemathematics.org/illustrations/368)*,* Illustrative Mathematics  **WHAT:** In this task, students are again given a linear and exponential model to compare F.LE.1. This time, however, the base of the exponential function is greater than but very close to 1 so the exponential function does not grow as quickly as seen in [Exponential Growth Versus Linear Growth I](https://www.illustrativemathematics.org/illustrations/366).  **WHY:** The purpose of this task is to provide students with a linear and exponential model where, initially, it does not look like the exponential model will ever surpass the linear model. Students must use careful examination, precision and repeated reasoning of both linear and exponential models to explain how the exponential function will surpass the linear model in this situation F.LE.3 MP.6 MP.8. |
| **Sample Activity 3.5** | [*I Remember*](http://mathalicious.com/lessons/i-remember)*,* Mathalicious  **WHAT:** In this task, students investigate and construct a model for how memory loses its fidelity as the number of remembrances increases. Students create and examine both linear and exponential models MP.4 and use these to determine when a memory becomes semi or unreliable F.LE.2.  **WHY:** The purpose of this task is to provide students an opportunity to construct linear and exponential functions given information from a real-world context. Students demonstrate their understanding of of writing exponential functions by creating a function based on given information and then demonstrate application of exponential and linear functions by comparing their three models. Students are not explicitly told how to compare the three models so they must use appropriate tools in order to create and justify their ideas MP.3 MP.5. |
| **Sample Activity 3.6** | [*Equal Differences Over Equal Intervals 1*](https://www.illustrativemathematics.org/illustrations/350) *&* [*Equal Factors over Equal Intervals*](https://www.illustrativemathematics.org/illustrations/363), Illustrative Mathematics  **WHAT:** In these tasks, students look at the change over given intervals in linear and exponential functions. Students should be very familiar with linear change being an equal difference over equal intervals so the first task is used to clarify notation and linear change. The second task has students look at change in an exponential function, noticing that the change is an equal factor over a given interval rather than an equal difference as is in the linear case F.LE.1a. It is recommended to complete these problems successively to allow students a chance to notice the specific differences in the rate of change between linear and exponential functions.  **WHY:** Throughout this section, students have been making observations about the differences between linear and exponential functions. These problems serve as a way to formalize those observations and prove the differences in rate of change between linear and exponential functions MP.8. |
| **Focus Standards** | F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.  F.LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.  F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function~~. |
| **Mathematical Practices** | MP.1, MP.2, MP.3, MP.4, MP.5, MP.6, MP.8 |

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| **Section 4:** 2 - 3 days | **Percent Growth and Decay** |
| **Mathematical Goals** | Students will...   * Be able to apply exponential functions in both growth and decay situations F.LE.1c * Write the appropriate exponential function f(t) = a(1±r)^t, when given a starting value and and rate of change over time F.LE.2 * Identify the rate of increase or decrease given an exponential function F.LE.2 * Recognize that exponential functions have a constant percent rate per unit interval F.LE.1c * Write an exponential function in a real-world context F.BF.1a |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, student construct exponential functions when given an initial value and a growth/decay rate. Students recognize the rate of increase or decrease when given an exponential function and understand the meaning of this rate in the context of the problem. Students continue to build on the notation that an exponential function has a constant percent rate per unit interval. |
| **Sample Activity 4.1** | [*US Population 1790 - 1860*](https://www.illustrativemathematics.org/illustrations/354)*, Illustrative Mathematics*  **WHAT:** In this task, students continue to build on their understanding of the difference between linear and exponential models F.LE.1. Additionally, in this task, students begin to evaluate and construct an exponential growth function to model the given data, recognizing that constant percent rate per unit interval is modeled with an exponential function F.LE.1c MP.4.  **WHY:** The purpose of this task is to build on the understanding of the differences between linear and exponential functions while taking it a step further having students evaluate and construct an exponential model. |
| **Sample Activity 4.2** | [*Basketball Rebounds*](http://www.illustrativemathematics.org/illustrations/347), Illustrative Mathematics  **WHAT:** The Basketball Rebounds task uses the requirements set by the International Basketball Federation regarding the required rebound height of a basketball. They use this information to find a formula for the height of the ball, given the number of bounces. The third part of this task has students solve an exponential equation where logs are needed - it is recommended that the third part either be omitted as students have not yet seen logs OR the teacher can introduce technology that will solve the exponential equation without having to introduce logs.  **WHY:** This task provides a real-life context where students can use the given information to build an exponential function to model the bounce height of a basketball as a function of the number of bounces F.BF.1a F.LE.2. |
| **Sample Activity 4.3** | [*In the Billions and Exponential Modeling*](https://www.illustrativemathematics.org/illustrations/566), Illustrative Mathematics  **WHAT:** In this task, students model exponential growth with real data about global population. Students calculate the rate of change over various time periods and use this to construct an argument as to whether or not an exponential model is the best way to represent the data MP.3.  **WHY:** This problem provides students with an opportunity to demonstrate their understanding of an exponential function having constant percent rate per unit interval F.LE.1c in a real world context. |
| **Sample Activity 4.4** | [Moore's Law and Computers](https://www.illustrativemathematics.org/illustrations/1906), Illustrative Mathematics  **WHAT:** In this task, students are given information about hard disc capacity and how it has grown over the last several decades. Based on the given information, students write an exponential function model that fits the given data F.LE.2 MP.4. Students use their exponential model to make predictions about the future storage capacity of hard drives.  Part d. of this activity requires use of the natural log to solve. If students have not yet been introduced the the natural log, it is recommended that you skip this part or use this as an opportunity to teach the natural log in a real-world context.  **WHY:** The purpose of this task is to provide students with an opportunity to create an exponential model in a real-world context. Additionally, students must attend to precision when creating their model and using it to make their future predictions MP.6. |
| **Focus Standards** | F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.  F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.  F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| **Mathematical Practices** | MP.3, MP.4, MP.6 |

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| **Section 5:** 1 days | **Bringing it All Together** |
| **Mathematical Goals** | Students will...   * Write, evaluate and graph both linear and exponential models of depreciation F.LE.2 * Compare and contrast linear and exponential models F.LE.1 * Compare and contrast different exponential models of depreciation F.IF.8b * Calculate and interpret linear and exponential rates of change in a context F.LE.5 * Construct linear and exponential models A.CED.1 |
| **Narrative overview of section**  (and how the standards are achieved) | In this section, students bring together all that they have learned in this unit. Students understand how to write linear and exponential models when given initial and final values. They can use this information to compare the two models and understand the differences between the models. Additionally, students examine multiple exponential models and can thoughtfully and thoroughly compare and contrast the data from the different models. |
| **Sample Activity 5.1** | [*iPod dPreciation*](http://mathalicious.com/lessons/ipod-dpreciation)*,* Mathalicious  **WHAT:** In this task, students are given data about the cost depreciation of different iPod models. Based on the costs given, students construct both linear and exponential models A.CED.1 F.LE.2 MP.4 and analyze and discuss which model is most realistic MP.2. Students also compare and contrast the depreciation of a few different Apple products, understanding the different depreciation rates and how they relate to the others F.IF.8b.  **WHY:** The purpose of this task is to provide students with a real-world situation where both a linear or exponential model might be viable F.LE.1. Students demonstrate their understanding by examining and interpreting the models F.LE.5 in order to construct a viable argument about which model is more realistic. In the last part of the task, students demonstrate an understanding of analysis of multiple exponential models by explaining which model has best maintained its value MP.3. |
| **Focus Standards** | A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.  F.IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.  F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.  F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. |
| **Mathematical Practices** | MP.2, MP.3, MP.4 |

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| **Section 6:** 1 day | **Summative Assessment** |
| **Pre-Unit Assessment Targets** | Assess students’ ability to   * construct and compare linear and exponential functions given data F.LE.2 * recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another F.LE.1c * explain in words the similarities and differences of linear and exponential models F.LE.1a * distinguish, model and solve situations that can be modeled with linear functions and with exponential functions F.LE.1 |
| **Sample Activity** | [*Unit Assessment*](https://docs.google.com/a/hightechhigh.org/file/d/0B784Ztw6k5CsVUE4M1lXaDVzclk/edit) |

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|  | 1.1 | 2.1 | 2.2 | 2.3 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 4.1 | 4.2 | 4.3 | 4.4 | 5.1 |
| F.IF.7.e |  |  | **X** | **X** |  |  |  |  |  |  |  |  |  |  |  |
| F.LE.1 | **X** | **X** |  | **X** | **X** | **X** | **X** | **X** |  |  | **X** |  |  |  | **X** |
| F.LE.1a |  |  |  |  |  |  |  |  |  | **X** |  |  |  |  |  |
| F.LE.1c |  |  |  |  |  |  |  |  |  |  | **X** |  | **X** |  |  |
| F.LE.2 | **X** |  |  |  |  |  |  |  | **X** |  |  | **X** |  | **X** | **X** |
| F.LE.3 |  |  |  |  |  |  | **X** | **X** |  |  |  |  |  |  |  |
| F.LE.5 |  | **X** |  | **X** |  |  |  |  |  |  |  |  |  |  | **X** |
| F.BF.1a |  |  |  |  |  |  |  |  |  |  |  | **X** |  |  |  |
| A.CED.1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **X** |
| F.IF.8b |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **X** |

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|  | 1.1 | 2.1 | 2.2 | 2.3 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 4.1 | 4.2 | 4.3 | 4.4 | 5.1 |
| MP.1 |  |  |  |  | **X** |  |  |  |  |  |  |  |  |  |  |
| MP.2 |  |  |  |  | **X** |  |  |  |  |  |  |  |  |  |  |
| MP.3 |  |  |  | **X** | **X** |  |  |  | **X** |  |  |  | **X** |  | **X** |
| MP.4 | **X** | **X** | **X** | **X** | **X** |  |  |  | **X** |  | **X** |  |  | X | **X** |
| MP.5 |  | **X** |  |  |  |  |  |  | **X** |  |  |  |  |  | **X** |
| MP.6 |  |  | **X** |  |  |  |  | **X** |  |  |  |  |  | X |  |
| MP.7 | **X** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MP.8 |  |  |  |  |  |  |  | **X** |  | **X** |  |  |  |  |  |